Structural Analysis of an Experimental Cable-Supported Air-Inflated Greenhouse

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THE utilization of air-inflated, double layer plastic greenhouses in intensive agricultural production is increasing rapidly. A major reason for the widespread popularity of these structures is their relatively low cost. Recent improvements in their design have demonstrated a trend towards the simplification and minimization of structural members in the framework and towards simpler schemes for attaching the plastic cover. The concept of using air under low pressures less than 6.3 mm (0.25 in.) of water to separate the film layers has enabled the development of efficient schemes for attaching the plastic cover (Roberts et al. 1972; Roberts and Mears 1969).

Any reduction or simplification of structural members in a greenhouse can have a twofold benefit. Shading in the house is diminished and the initial cost of structure is reduced. Therefore, a careful analysis of the various structural components is essential if a minimal design is to be achieved. Fortunately, relatively small factors of safety can be justified for these components as they are loaded by the plastic film covers which will fail before the framing if the design is adequate. Also, a failure of the cover or a partial failure of structural components does not pose as serious a safety hazard as in other types of buildings nor are the economic consequences as severe.

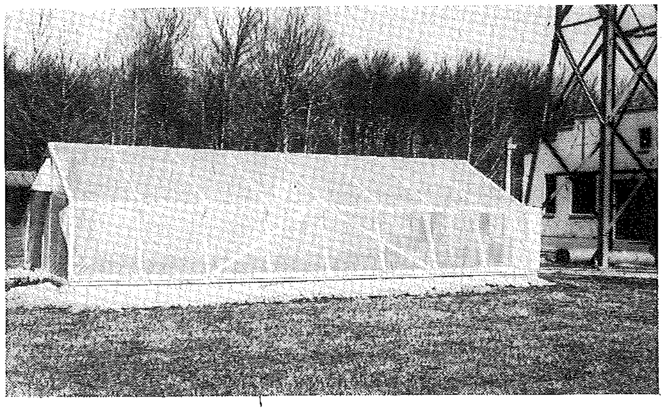

FIG. 1a Typical slant-leg rigid frame greenhouse.

Recently the authors have reviewed various types of air-inflated and air-supported greenhouses (Roberts et al. 1972). The development of an experimental, modular, cable-supported greenhouse was reported. An analysis of the loads, stresses and deformations of the plastic cover and of the various structural components are discussed here.

In previous work the relationship between the deflection of rectangular sections of plastic films from the plane of support, the internal air pressure, the dimensions of the section and the film stress in the plastic has been presented (Roberts and Mears 1969). This analysis is based upon the assumption that deflections of the film from the plane of support are relatively small, which is the case in the type of structure shown in Fig. 1-a. When the length of the rectangular plastic film is at least three times the width, the deflection of the plastic film midway between the sides of the rectangular section is given by:

$$z = \frac{pa^2}{2S} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad [1]$$

where

z = Deflection, cm (in.)
p = Pressure between the films, Pa (psi)
a = One-half the short side of the rectangle, cm (in.)
S = Stress in the film, N per lineal cm (lb per lin. in.)

In the case of the cable-supported greenhouse, cables 183 cm oc, shown in Fig. 1-b, the outer roof sections are supported in a rectangular plane 3.66 m by 14.6 m (12 ft by 48 ft).

In this structure the maximum deflection of the upper film of 46 cm (18 in.) is significant when compared to the dimension "a" which is 183 cm (72 in.). For this situation a new analysis of the relationship between dimensions, deflection, pressure and stress in the film is needed. The shape of a roof section can be approximated by a long section of a cylinder with fixed ends. The geometry of the section and identification of the forces, stresses and strains

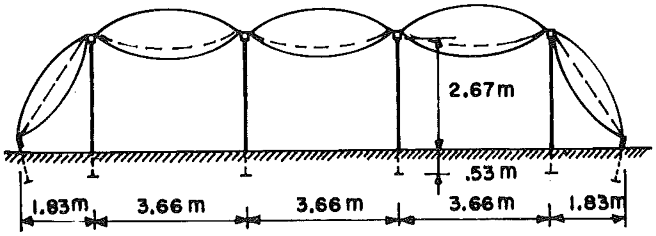

FIG. 1b Experimental cable supported air-inflated greenhouse (14.6 m x 14.6 m).

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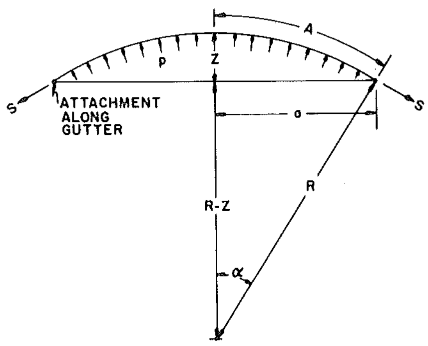
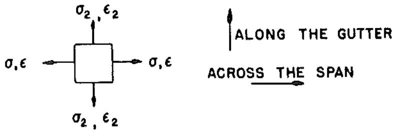


CROSS SECTION OF ROOF, GUTTER SPACING = 2a



ELEMENT OF PLASTIC FILM

FIG. 2 Geometry and stresses in a roof section of the cable support house.

TABLE 1. MECHANICAL PROPERTIES OF POLYETHYLENE TUBE USED TO COVER CABLE GREENHOUSE.

Strain direction	Tensile strength, M Pa	Yield strength, M Pa	Secant* modulus, G Pa
Longitudinal	8.49	6.62	0.330
Longitudinal	8.90	4.76	0.238
Longitudinal	8.27	6.01	0.300
Longitudinal	9.05	7.03	0.351
Transverse	7.45	5.79	0.289
Transverse	8.27	5.79	0.289
Averages	8.40	6.00	0.300

*Secant modulus based on 2 percent yield.

are given in Fig. 2. The symbols not previously defined are:

t = thickness of the plastic film, cm (in.)
R = Radius of curvature of roof section after inflation, cm (in.)
A = One-half the arc length of the plastic film between the gutters after inflation, cm (in.)
A' = One-half the arc length of the plastic film between the gutters before inflation, cm (in.)
a = The angle subtended by the arc A, radians
σ = Tensile stress in the plastic film perpendicular to the gutters after inflation, Pa (psi)
ε = Strain in the plastic film perpendicular to the gutters after inflation, cm/cm (in./in.)
σ_2 = Tensile stress in the plastic film parallel to the gutters after inflation, Pa (psi)
ε_2 = Strain in the plastic film parallel to the gutters after inflation, cm/cm (in./in.)
E = Modules of elasticity of the plastic film, Pa (psi)
μ = Poisson's ratio

When applying the plastic film to the frame some slack can be left so A′ is somewhat greater than a. The inflation pressure, p, can be held at any desired level. The relationship between these variables, the geometry of the frame, the deflection of the film and the stress in the film has been developed (Kim 1972). The analysis is based on the assumption that the film is under plane stress and that there is no strain parallel to the long axis of the cylinder away from the ends of the building. The equations resulting from this analysis which can be used in design are:

$$S = pR \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

$$S = \sigma t \quad \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

$$A = A' \left[1 + \frac{\sigma}{E}(1 - \mu^2)\right] \quad \ldots\ldots\ldots [4]$$

$$\frac{ap}{\sigma t} = \sin\left[\frac{A'p}{t}\left(\frac{1}{\sigma} + \frac{1 - \mu^2}{E}\right)\right] \quad \ldots . [5]$$

For this structure the span of 3.66 m (12 ft), a = 183 cm (72 in.), was selected to fit available lumber sizes and widths of plastic tubing (28 ft circumference). The vertical deflection of 45.6 cm (18 in.) was selected to give the desired shape. Having picked these two parameters, the geometry shown in Fig. 2 is used to compute the radius of curvature R and the final arc length of the plastic film between the gutters. In this case: R = 389 cm (153 in.), A = 190 cm (74.96 in.), 2A = 380 cm (149.92 in.). Next the design operating pressure p is selected, in this case 10 mm (0.4 in.) of water pressure. From equation [2 pressure. From equation [2] the lineal stress S in the plastic film can be calculated to be 3.85 N/cm (2.2 lb/in.). The thickness of the film used was 5 mil (0.005 in.).

The greenhouse grade polyethylene was nominally 4 mil, but by actual measurement the mean thickness was found to be 0.126 mm (4.97 mil) with a variance of 0.175 mm (0.69 mil). From equation [3] the stress in the film σ is 3.03 x 10^6 Pa (440 psi). Since this stress is well below the published yield stress of polyethylene film (3,100 to 5,500 psi) (Nielson 1962), nominal 4-mil film is acceptable. Several strips were cut from the tube used to cover the house and tested in a universal testing machine. The results are presented in Table 1. Calculations for the properties were based on an average of 12 thickness measurements.

Equation [4] can now be used to compute the arc length A′ before inflation. Assuming the elastic modulus E is 2.76 x 10^8 Pa (40,000 psi) (determined by actual tests) and Poisson's ratio μ is 0.38 (Roberts et al. 1972) the computation gives A′ = 189 cm (74.26 in.), 2A = 378 cm (148.52 in.). As the span, 2a, is 366 cm (144 in.) in the top layer of plastic when the gutter connections are made fast, assuming that there is no slippage of this connection when the roof is inflated, there should be 12 cm (4.52 in.) slack. Application of 10 mm (0.4 in.) of water pressure will stretch the film an additional 2.54 cm (1 in.) resulting in a final rise z of 45.6 cm (18 in.). Equation [1] predicts a stress of 2.86 x 10^6 Pa (415 psi). As the radius of curvature is increased, the relationships of equation relationships of equation [1] converge to agree wih the relationships developed above.

Small changes in pressure will have a relatively small effect on R and z, but the effect on stress in the plastic will be almost proportional to the change in the pressure. For example, in the preceding analysis

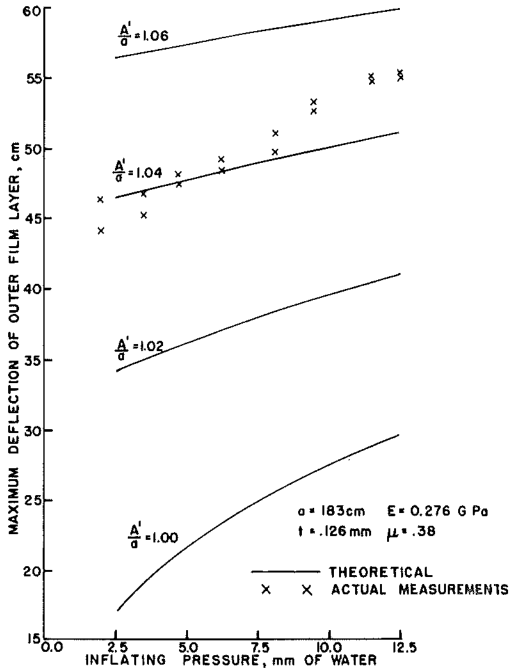


FIG. 3 Relationship between the deflection of the outer-film layer and the pressure between the layers.

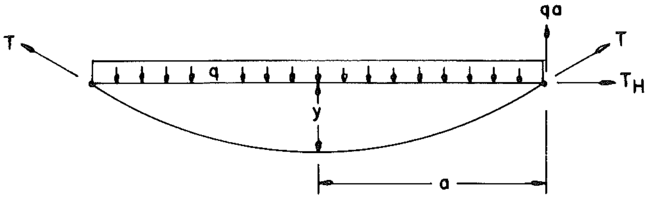


FIG. 4 Loading, deflection and forces in the cable.

the maximum anticipated operating pressure of 10 mm (0.4 in.) of water pressure was selected for design purposes. However, the fan which inflates the air space between the roof layers is usually operated at 5 mm (0.2 in.) of water. Equation [5] can be used to compute the stress, which is 1.61 x 10^6 Pa (234 psi). Under these conditions the radius of curvature R is 411 cm (162 in.) and the deflection z is 43 cm (16.9 in.).

Fig. 3 shows the relationship between the deflection of the outer film layer and the pressure between the layers for several ratios of A′ to a. The curves represent predicted relationships and the X's represent actual measurements made on the building. In applying the plastic, slack was allowed in the span for actual ratio A′/a = 1.04. The constants used for the calculations in this figure are those used in the preceding calculation. There is close agreement between the measured and the predicted values.

TABLE 2. ACTUAL MAXIMUM DEFLECTIONS AND STRESSES FOR THE INNER FILM LAYER FOR VARIOUS PRESSURES.

Pressure mm of water	Midbay deflection, cm	Calculated stress, M Pa
0	8.9	0
1.52	11.2	0.40
3.12	11.9	0.76
5.46	12.4	0.13
7.11	12.7	1.63
8.64	12.9	1.94
10.16	13.2	2.24
12.95	13.6	2.76

Having selected all of the parameters associated with the upper layer of the roof, it is necessary to next consider the lower roof layer. As the roof is applied as a single tube clamped along the gutters, the thickness of the lower layer is the same as the upper layer. Furthermore, as the span between the gutters is twice that of the cable supports of the lower layer, it is apparent that both the deflections and stresses in the lower layer will be much less than in the upper layer. As the deflections are not as great as in the upper layer, equation [1] can be used to relate the inflation pressure, the stress in the film and the deflections below the plane of the supporting cables.

Actual measurements of the deflection of the lower layer in one of the bays were made for varying pressures and the results are presented in Table 2. The stresses are 0.9 of those calculated by equation [1]. The factor of 0.9 allows for the support to the film from the long dimension for a 1.83 m by 3.55 m (6 ft by 12 ft) rectangle (Roberts and Mears 1972). Note that even though the maximum film deflection measured at the center of the bay is less than a third that of the upper film layer, the stresses are significantly less, due to the fact that the film span between cables is half that of the upper film layer.

The design of the supporting cables can now be considered. Each cable span is loaded by the plastic film and is restrained by a tensile force T at each end. An analysis which determins the relationships between the geometry of the roof, the pressure between the roof layers, the tension in the cable, and the maximum deflection of the center of the cable has been developed (Kim 1972).

A section of the cable is shown in Fig. 4. The load q in N per cm (lb per in.) of cable can be computed by multiplying the pressure, p, between the layers by the cable spacing. The other terms in the figure are:

a = One-half the span length, cm (in.)
y = The midspan deflection, cm (in.)
T = The tensile force in the cable, N (lb)
T_H = The horizontal component of T

For the purpose of this analysis, it was assumed that y was small compared to a. In practice this can be achieved by placing an initial tension force T_O on the cable before the load q is applied. The tension force in the cable required to produce a unit increase in length is K, N/cm (lb/in.). From the analysis the following equations were developed:

$$T = qa\ [1 + \frac{a^2}{4y^2}]^{1/2} \quad \ldots\ldots\ldots\ldots\ [6]$$

and

$$T_O = \frac{qa^2}{4y}\left[1 + \left(\frac{4y^2}{a^2}\right) + 1^{\ 1/2}\right] - 2/3\ K\ \frac{y^2}{a^2} \quad \ldots\ldots\ldots\ldots\ [7]$$

Equation [7] gives the initial tension T_O which should be applied to the cable to limit the deflection y to a desired amount once the design parameters have been selected.

Consider as an example that it is desired that the maximum cable deflection y be 5 cm (2 in.) when the

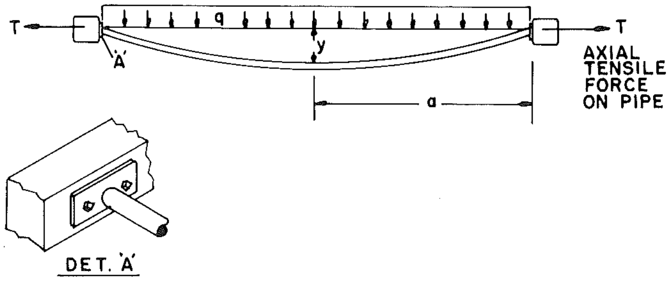


FIG. 5 Loading and deflection of pipe roof support.

pressure between the layers is 5 mm (0.2 in.) of water, the normal operating pressure in this house. The cable spacing of 1.83 m (6 ft) means that q will be 92 N per M (0.52 lb/in.) of cable length. For this house a 7.94 mm (5/16 in.) diameter plastic coated steel cable was used and its initial "spring constant" K was measured and found to be 58,000 N per cm (33,000 lb/in.). Using these values in equation [7] it is found that the initial tension on the cable T_0 must be 2927 N (658 lb). Using equation [6] the maximum cable tension T is found to be 3007 N (676 lb) when the pressure is applied.

An alternative to the use of a cable to support the inner layer of plastic was also developed and used. Sections of 3/4 in. standard pipe were fastened between the gutters as shown in Fig. 5. The load q is due to the air pressure between the film layers and the weight of the pipe itself which is significant. The ends of the pipe are welded to a flat plate which is fastened to the gutter by two lag screws as shown in detail A of Fig. 5. For the purpose of design, it was assumed that this joint was completely rigid, i.e., the pipe could be considered to have built-in ends. Also, the gutters were assumed to be fixed in place so that there could be no displacement of the ends. Therefore, an axial force T can be developed in the pipe. The following equations for a tie rod with built-in ends and combined axial and uniform lateral loading are taken from Timoshenko (1956) or developed from that analysis (Kim 1972). The deflection y_0 at the center can be approximated by:

$$y_0 = \frac{\delta_0}{1 + \frac{\alpha}{4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [8]$$

where

$$\delta_0 = \frac{1}{24}\,\frac{qa^4}{EI} \quad \ldots\ldots\ldots\ldots\ldots\ldots [9]$$

E is the elastic modulus of steel, I is the modulus of the pipe section and:

$$\alpha = \frac{4T\,a^2}{\pi^2 EI} \quad \ldots\ldots\ldots\ldots\ldots\ldots [10]$$

In order to solve for the tensile force T, the term *α* is computed by use of the following:

$$\alpha\left(1 + \frac{\alpha}{4}\right)^2 = \frac{\delta_0^2\, a'}{4I} \quad \ldots\ldots\ldots\ldots [11]$$

where a′ is the cross-sectional area of the pipe.

Using this analysis, a design pressure of 10 mm (0.4 in.) of water and the properties of standard 3/4-in. pipe, the predicted midspan deflection of the pipe is found to be 2.1 cm (0.825 in.) and the maximum combined stress due to the bending moment and the axial tensile force T is predicted to be 1.72 x 10^8 Pa (25,000 psi). This design stress is somewhat higher than the 1.38 x 10^8 Pa (20,000 psi) usually used. However, in the actual structure the end conditions specified in the analysis cannot be met completely. There can be some rotation at the end connection and it is possible for the outer gutters to move towards the center due to stretch in the cable leading to the side anchors. Both of these factors will tend to increase the deflections and reduce the stresses. In order to evaluate this situation the midspan deflection was recomputed based on three other end conditions: (a) no lateral deflection but hinged ends, (b) rigid ends but no lateral force T, and (c) hinged ends and no lateral force T. This last end condition predicts the largest deflection and smallest stress while the other two are intermediate when compared to the completely restrained condition. Actual measurements made on the structure show that deflections are less than those predicted by end conditions (c) but more than those predicted by the other end conditions. A wider house would be expected to have less lateral deflection of the gutters, especially in the interior spans, and the pipe to gutter connections could be made more rigid.

Actual measurements of the cable and pipe supports were made under varying pressure conditions and are presented in Table 3. The diagonal cables tying the outer gutters to the anchors in the ground are spaced every 1.83 m (6 ft) and are in line with either cable roof supports or pipe roof supports. Tensile loads in cables for both cable and pipe roof supports are presented in Table 3. The midspan deflections of both cable and pipe roof supports are given for each of the three bays in the greenhouse.

Cable support deflections are significantly less than those calculated according to the preceding analysis. This is most likely due to the tendency of cable to stiffen under use. (After application of a tensile load the cable is permanently elongated.) Field measurements were taken after the

TABLE 3. TENSILE FORCES IN CABLES AND MAXIMUM DEFLECTIONS OF CABLE AND PIPE ROOF SUPPORTS FOR VARIOUS PRESSURES.

Inflating pressure, mm of water	Tension exerted on side cable, N		Maximum deflection of roof cables, cm			Maximum deflection of roof pipes, cm		
	Cable row	Pipe row	Left bay	Center bay	Right bay	Left bay	Center bay	Right bay
0	1138*	490	0.51	1.0	0.25	2.4	2.8	1.5
1.52	1383	892	3.6	5.0	3.6	3.3	4.3	2.5
3.12	2118	1598	5.0	6.7	5.3	4.6	5.8	3.8
5.46	2608	2432	6.4	8.1	6.5	5.6	7.6	4.8
7.11	2981	2569	7.1	9.1	7.4	6.4	8.8	5.5
8.64	3609	2864	7.9	10.4	8.1	7.1	9.9	6.1
10.2	3756	3462	8.4	10.9	8.5	7.9	10.7	6.6
13.0	4158	3962	9.4	12.2	9.4	9.1	12.6	7.2

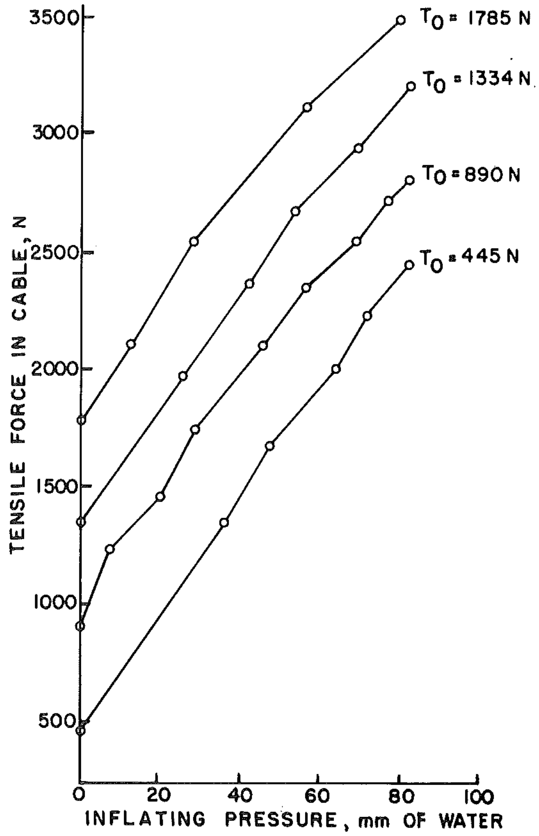


FIG. 6 Relationship between the tensile force in cable and the pressure between the film layers (actual measurements).

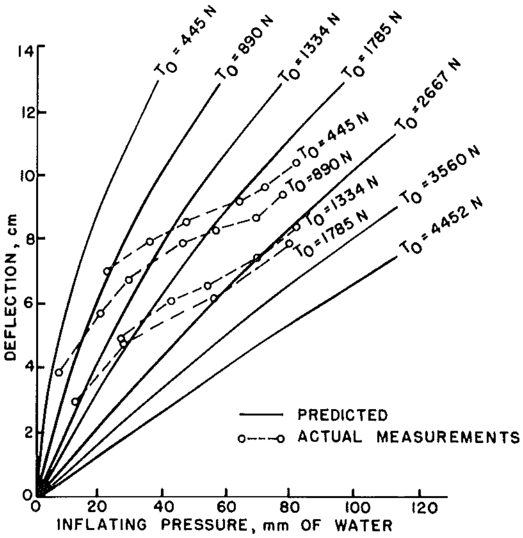


FIG. 7 Relationship between the cable deflection and the pressure between the film layers.

house had been erected for some time and slack had been taken out of the cables several times. The stiffness of the cable, K = 58,000 N cm (33,000 lb/in.), used in the calculations, was based on tests of new cable.

Data taken on the relationship between cable stress and inflating pressure for several initial tensions are presented in Fig. 6. The cable tension increases approximately 623 N (140 lb) for each 2.54 mm (0.10 in.) of water pressure increase, somewhat less than the 832 N (187 lb) predicted. This is due to stretch in the diagonal side cable and resultant movement of the gutters. In a wider house the actual load would be expected to more closely approach the theoretical.

The relationships for cable deflections at various pressures and for several initial tensions are presented in Fig. 7. From this it can be seen that an actual deflection of 5 cm (2 in.) under an applied pressure of 5 mm (0.2 in.) of water could be obtained with only a 1779 N (400 lb) T_O value as opposed to the theoretical value of 2927 N (658 lb). Thus it is seen that the formulas relating to the cable design and utilizing cable stiffness are conservative for both load and deflection.

The wind loads on the building have also been analyzed on a preliminary basis. The results of the study so far indicate that more work needs to be done to understand the effects of wind on plastic-covered greenhouses, especially with regard to the uplift forces on the roof.

Using the method of the Rutgers Farm Building Standard (Reed 1965) for calculating wind load pressures and assuming 96.1 km/h (60 mi/h) wind load, Q is 335 N/m^2 (6.97 lb/ft^2) and the following wind loadings are obtained:

Vertically up on the roof: 257 N/m^2 (5.35 lb/ft^2)

On the windward vertical end: 335 N/m^2 (6.97 lb/ft^2) inward

On the leeward vertical end: 194 N/m^2 (4.04 lb/ft^2) outward

On the sloping side parallel to wind directions: 194 N/m^2 (4.04 lb/ft^2) outward

These loadings are based on the conventional structural shape factors.

If these wind loads were applied to the structure, there would be an increase in the stress in the outer layer of plastic equivalent to that produced by a static pressure between the layers of 2.61 cm (1.03 in.) of water which would lift the building from its foundation or cause tensile failure of the plastic. However, these effects have not been observed, either on large buildings or on the experimental house which did experience 100 km/h (60 mi/h) winds.

The most probable explanation for this apparent discrepancy between predictions and experience is that the predicted wind loads are much too high. In high winds one can observe that the shape of the flexible outer layer changes continuously in response to locally varying wind pressures. It would appear that this shape always tends to be such that the wind pressures normal to the plastic surface are minimized, according to Hamilton's principle. Therefore, the shape factors to be used in predicting wind loads on inflated plastic buildings should be much smaller than for rigid buildings of similar shape. More work is needed to measure wind loads under various conditions so that more realistic criteria can be established for the design of these structures.

In the event that the snow load on the roof were to exceed the equivalent of 10 mm (0.4 in.) of water, some alteration of the design or action to remove the snow must be instituted. One solution would be to increase the air pressure between the layers to support the weight of the snow. This would require designing the structure for the indicated pressure. Another proposed solution is to pressurize the interior of the building forcing the inner roof layer up against the outer layer. This would essentially double the strength of the roof as the internal pressure could be doubled

(Continued on page 924)

Analysis for Air-Inflated Greenhouse

(Continued from page 919)

for a given film stress. More important, it would provide for more rapid heat transfer through the roof, increasing the rate at which the snow could be melted. Not only does this remove the weight from the structure, but it also eliminates the snow cover which would otherwise shade the house following the storm. Assuming a heat transfer coefficient through the roof of 5.67 W/m²K (1 BTUh/ft² °F) and an interior temperature of 23.9 °C (75 °F), the snow could be melted at a rate of about 1.5 mm (0.06 in.) of water per hour, i.e., about 15 mm of snow per hour. In order to do this, it is necessary that the ventilation and heating systems be designed so that internal house pressure can be maintained with the heat on. The internal air pressure must be reduced as the snow load is melted off to avoid lifting the entire house.

During the winter of 1973-1974 actual snow melting rates were determined. It was found that the actual heat transfer coefficient averaged 5.45 W/m²K (0.96 BTUh ft² °F). The major resistance to heat transfer (from the warm air in the house to the melting snow) is the convective film coefficient on the inside of the plastic.

The gutters used were analyzed and it was found that the 4 in. x 4 in. lumber used in this experimental design is more than adequate for the maximum loadings. Further work will be done on the design of a rolled steel or extruded aluminum gutter and a fixture for connecting the gutter to the cables and the poles.

The apparent advantages of this experimental structure are:

1 The minimization of the structural components reduces shade within the greenhouse.

2 The modular design with simple components reduces labor and material costs for the structural frame.

3 The structural components can provide support for crops which require it.

4 The plastic cover can be easily and rapidly applied or removed without disturbing the crop planted within.

Replication of the 3.66 m (12 ft) modular dimension can result in large areas being covered simply and economically. Bays 29.3 m (96 ft) long can utilize 30.5 m (100 ft) rolls of plastic tubing and any number of bays can be placed side by side.

The major conclusions of this work are:

1 The formulas developed for analyzing stresses in the plastic film can be used for design purposes. When deflections are relatively small, the simplified formula (1) can be used.

2 The analysis of the cable roof support system is conservative and acceptable for design. It would be helpful to know the effect of the apparent stiffening of cable with use on this analysis.

3 In designing pipe roof supports, the formulas for rigid end conditions with no lateral movement give a conservative stress analysis. The simply supported end condition will predict conservative estimates of the deflections. These two taken together will result in acceptable designs.

4 Wind loads on plastic-film-covered greenhouses are apparently significantly less than those calculated by the method that is used for farm buildings with rigid walls and roofs.

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